MEETING #3

HADAMARD GATE:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \in O(2) \subseteq U(2)$$

$|0\rangle \longleftrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad |1\rangle \longleftrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|0\rangle, |1\rangle$

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) =: |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) =: |-\rangle$$

$$\boxed{\begin{aligned} X &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ X|\pm\rangle &= \pm|\pm\rangle \end{aligned}}$$

$$\mathcal{H}_n = \underbrace{\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \dots \otimes \mathbb{C}^2}_{n\text{-FOLD}}$$

$x \in \mathbb{F}_2^n$

$$|x\rangle = |x_0\rangle \otimes |x_1\rangle \otimes \dots \otimes |x_{n-1}\rangle$$

$n = 3$

$x = (1, 0, 1)$

$|x\rangle = |1\rangle \otimes |0\rangle \otimes |1\rangle$

$$|S\rangle := \frac{1}{\sqrt{2^n}} \sum_{x \in \mathbb{F}_2^n} |x\rangle$$

1. $\mathbb{P}(\text{MEASURING } x \text{ IN } |S\rangle) = \frac{1}{2^n}$

2. $|S\rangle = H^{\otimes n}|0\rangle = (H \otimes H \otimes \dots \otimes H)(|0\rangle \otimes |0\rangle \otimes \dots \otimes |0\rangle$

$= (H|0\rangle) \otimes H|0\rangle \dots$

$= |+\rangle \otimes |+\rangle \otimes \dots \otimes |+\rangle$

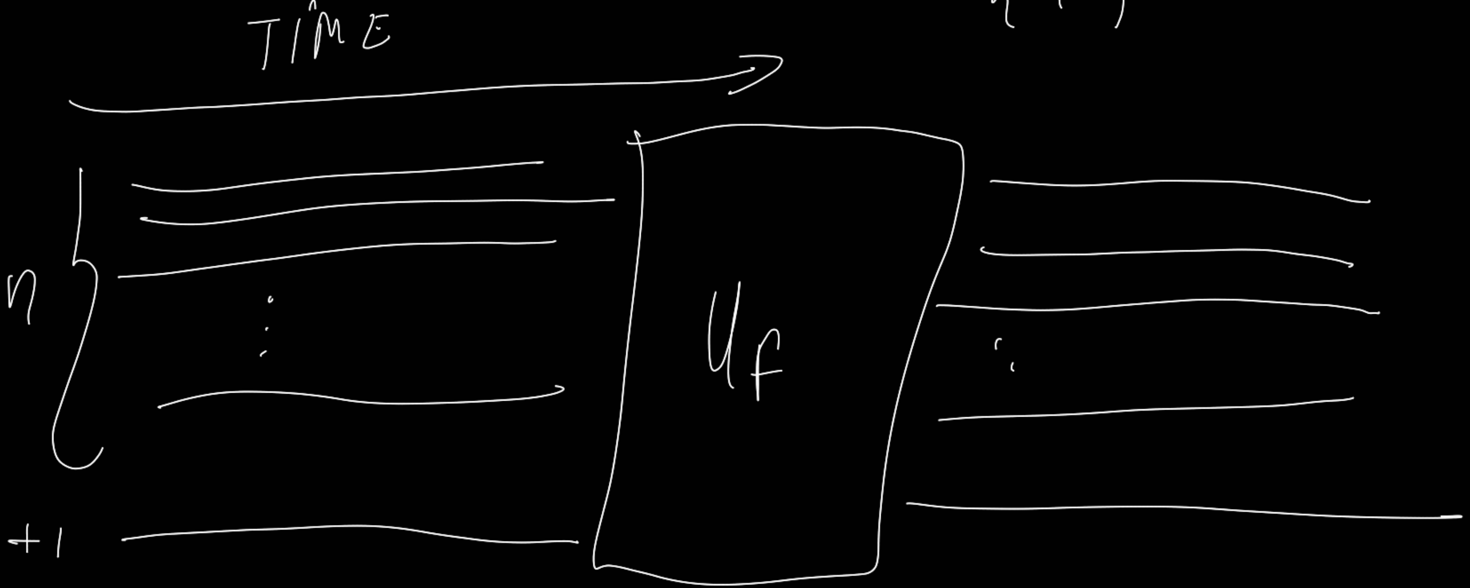
ORACLES / QUERIES :

ORACLE: "BLACK BOX" "QUANTUM CIRCUIT"

EX: $f: \mathbb{F}_2^n \to \mathbb{F}_2$

$n = 3$ $\quad f(x_0, x_1, x_2) = y \in \{0,1\}$

$x_0, x_1, x_2 \in \{0,1\}$



$U_F|x\rangle|y\rangle := |x\rangle|y \oplus f(x)\rangle$

$x \in \mathbb{F}_2^n \quad y \in \mathbb{F}_2$ $\quad \oplus$: MOD 2

ADDITION

QUERY (-COMPLEXITY)

GIVEN AN Q-ALGO, W/ AN ORACLE $\mathcal{U}$

QUERY-COMPLEXITY = # OF TIMES $\mathcal{U}$ NEEDS TO RUN

DEUTSCH-JOZSA :

$f: \mathbb{F}_2^n \longrightarrow \mathbb{F}_2$ & I HAVE ACCES TO U_F

& f IS PROMISED TO BE EITHER:

I. CONSTANT

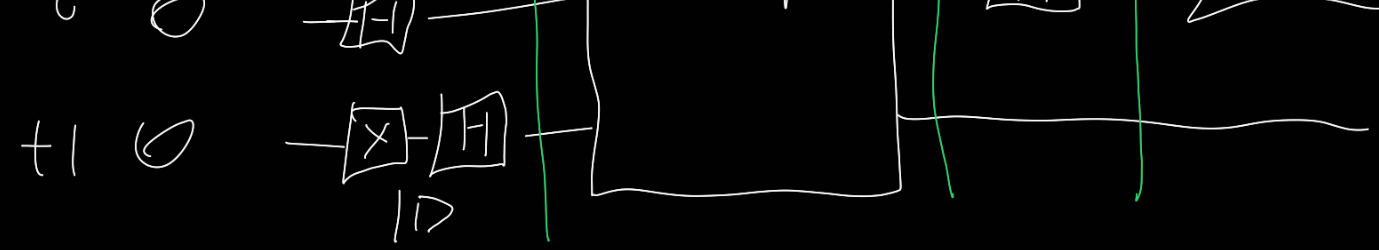
II. „BALANCED"

$$|f^{-1}(0)| = |f^{-1}(1)| = 2^{n-1}$$

Q: is f TYPE I OR II?

CL-A: CLASSICALLY NEED TO CHECK $f(x)$ FOR $2^{n-1}+1$ VALUES OF $x \in \mathbb{F}_2^n$

Q-A



CLAIM: YOU MEASURE $\underbrace{00\ldots0}_{n}$ IF & ONLY IF f IS CONSTANT!

QUERY-COMP $= \Theta(1)$

PROOF:

AT 1.

$$|0\rangle_n|0\rangle \to |S\rangle_n|-\rangle = \frac{1}{\sqrt{2^n}}\sum_{x\in\mathbb{F}_2^n}|x\rangle|-\rangle$$

AT 2.

$$\xrightarrow{U_f} \frac{1}{\sqrt{2^n}}\sum_{x\in\mathbb{F}_2^n} U_f|x\rangle|-\rangle =$$

$|x\rangle|-\rangle \quad f(x)=0$

$|x\rangle|-\rangle \quad f(x)=1$

$(-1)^{f(x)}|x\rangle|-\rangle$

$$= \frac{1}{\sqrt{2^n}} \sum_{x \in \mathbb{F}_2^n} (-1)^{f(x)} |x\rangle |-\rangle$$

$$\xrightarrow{H^{\otimes n}} \frac{1}{2^n} \sum_{x, y \in \mathbb{F}_2^n} (-1)^{x \odot y + f(x)} |y\rangle |-\rangle$$

<u>HW</u>

$$\boxed{H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \mathbb{F}_2^n} (-1)^{x \odot y} |y\rangle}$$

$$x \odot y = \sum_{a=0}^{n-1} x_a y_a \text{ MOD } 2$$

FINAL STATE:

$$|\psi\rangle = \frac{1}{2^n} \sum_{x,y \in \mathbb{F}_2^n} (-1)^{x \odot y + f(x)} |y\rangle$$

1. $f(x) = \text{CONST} = z \in \{0,1\}$

$$|\psi\rangle = \frac{(-1)^z}{2^n} \sum_{x,y} (-1)^{x \odot y} |y\rangle$$

$$\mathbb{P}(\text{MEASURING } \overbrace{0 \ldots 0}^{y}) =$$

$$\left| \frac{(-1)^z}{2^n} \sum_{x \in \mathbb{F}_2^n} \overbrace{(-1)^{x \odot 0}}^{1} \right|^2 = 1$$

$(y = 0)$

2.

$$\mathbb{P}(\ldots) =$$

$$= \left| \frac{1}{2^n} \sum_{x \in \mathbb{F}_2^n} (-1)^{f(x)} \right|^2 = 0$$

f IS BALANCED

$\boxtimes$